## ECS 315: In-Class Exercise \# 1

## Instructions

Separate into groups of no more than three persons.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.


1. ( 6 pt) A fair coin is flipped eight times. The results are:

$$
\begin{aligned}
& \text { Recall that, for the firs } n \text { trials } \\
& R(A n)=\text { trials that } A \text { occurs }
\end{aligned}
$$

THHTTHHT.
Let $A$ be the event that heads occurs.
Let $R(A, n)$ denote the relative frequency of event $A$ for the first $n$ flips.
Calculate $R(A, n)$ from $n=1$ to $n=8$. Write your answers in the form X.XX.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R(A, n)$ | $\frac{0}{1}=0.00$ | $\frac{1}{2}=0.50$ | $\frac{2}{3} \approx 0.67$ | $\frac{2}{4}=0.50$ | $\frac{2}{5}=0.40$ | $\frac{3}{6}=0.50$ | $\frac{4}{7} \approx 0.57$ | $\frac{4}{8}=0.50$ |

2. $(3 \mathrm{pt})$ List all of Dr.Prapun's office hours for this week.

Hint: Check Google Calendar on the course website. http://www2.siit.tu.ac.th/prapun/ecs315/index.html

3. (1 pt) (This one final task has to be worked on individually, not as a group.)

Use your own Line account to send
your student id, followed by your full name, and then your nickname inside the parentheses
into the ECS315 Line group.
Example: "50764555 Nadech Kugimiya (Barry)"

Remark: If you don't have a Line account, email the message as instructed above to prapun@siit.tu.ac.th.

## ECS 315: In-Class Exercise \# 2

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: $\mathbf{2 3} / \underline{\mathbf{0}} / 2018$ |  |  |  |
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| Name | ID |  |  |
| Prapun | 5 | 5 | $\mathbf{5}$ |
|  |  |  |  |
|  |  |  |  |

For each of the sets provided in the first column of the table below, indicate (by putting a $\mathrm{Y}(\mathrm{es})$ or an $\mathrm{N}(\mathrm{o})$ in the appropriate cells of the table) whether it is "finite", "infinite", "countably infinite", "uncountable". Explanation is not needed.
(a)
(b)
(c)
(d)
(e)

|  | Finite | Infinite | Countably Infinite | Uncountable |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbb{R}$ | $\mathbf{N}$ | $\mathbf{Y}$ | $\mathbf{N}$ | $\mathbf{Y}$ |
| $\{\pi, 2 \pi\}$ | Y | $\mathbf{N}$ | $\mathbf{N}$ | $\mathbf{N}$ |
| $[1,3] \cap[2,4]$ | $\mathbf{N}$ | $\mathbf{Y}$ | $\mathbf{N}$ | $\mathbf{Y}$ |
| $[1,2] \cap[3,4]$ | Y | $\mathbf{N}$ | $\mathbf{N}$ | $\mathbf{N}$ |
| The set of all real- <br> valued $x$ satisfying <br> $\sin (x)=x$ | $\mathbf{Y}$ | $\mathbf{N}$ | $\mathbf{N}$ | $\mathbf{N}$ |

Each circle above indicates the key answer for the corresponding part.
(a) We have seen in class that the set of all real numbers is uncountable (Ex. 2.18). Any uncountable set is infinite. Any infinite set is not finite.
Any uncountable set is not countable and therefore can not be countably infinite.
(b) The given set contains only two elements. Therefore, it is finite.

Any finite set can not be infinite, countably infinite, nor uncountable.
(c) The intersection gives $[2,3]$ which is an interval of positive length (Ex. 2.18). Therefore, it is uncountable. Because it is an uncountable set, the answers should be the same as part (a).
(d) The intersection gives empty set which is a finite set.

Because it is a finite set, the answers should be the same as part (b).
(e) There is only one solution: $\mathbf{x}=\mathbf{0}$. Therefore, the set is a singleton which is finite.

Because it is a finite set, the answers should be the same as parts (b) and (d).
Note that we can plot $\sin (x)$ and $x$ and see their intersection.
Even with lousy plots, one can tell that the number of solution is finite.


## ECS 315: In-Class Exercise \# 3

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: 28/ 0ع / 2018 |  |  |  |
| :---: | :---: | :---: | :---: |
| Name | ID ${ }_{\text {bex }}$ |  |  |
| Prapun | 5 | 5 | 5 |
|  |  |  |  |
|  |  |  |  |

1) A random experiment has 24 equiprobable outcomes:

$$
\Omega=\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x\}
$$

Let $A$ denote the event $\{a, b, c, d, e, f, g, h, i, j, k, l\}$, and let $B$ denote the event $\{i, j, k, l, m, n, o, p\}$. Determine the following:
a) $P(A)=\frac{|A|}{|\Omega|}=\frac{12}{24}=\frac{1}{2}$
b) $P\left(A \cup B^{c}\right)=\frac{\left|A \cup B^{c}\right|}{|\Omega|}=\frac{|\Omega|-\left|\left(A \cup B^{c}\right)^{c}\right|}{|\Omega|}=1-\frac{\left|A^{c} \cap B\right|}{|\Omega|}=1-\frac{|B \backslash A|}{|\Omega|}=1-\frac{| | m, n, 0, p\} \mid}{24}$

$$
=1-\frac{4}{24}=\frac{5}{6} \approx 0.8333
$$

2) Consider a random experiment whose sample space is $\{a, b, c, d\} \quad\left|A \cup D^{\delta}\right|=20$ with outcome probabilities $0.2,0.2,0.3$, and 0.3 , respectively. $\quad P\left(A \cup B^{2}\right)=\frac{\left|A \cup B^{C}\right|}{|\Omega|}=\frac{20}{24}=\frac{5}{6}$
Let $A=\{a, b, c\}, B=\{c, d\}$, and $C=\{a, c\}$.
Find the following probabilities.
a) $P(A)=\mathbb{P}(\{\mathbf{a}, \mathbf{b}, \mathbf{c}\})=\mathbf{P}(\{\mathbf{a}\})+\mathbb{P}(\{\mathbf{b}\})+\mathbb{P}(\{\mathrm{c}\})=0.2+\mathbf{0 . 2}+\mathbf{0 . 3}=\mathbf{0 . 7}$
b) $P(A \cap B)=P(\{c\})=0.3$
c) $P\left(B^{c}\right)=P(\{a, b\})=P(\{a\})+P(\{b\})=0.2+0.2=0.4$
d) $P(A \cup B)=P(\{a, b, c, d\})=P(\{a\})+P(\{b\})+P(\{c\})+P(\{d\})=0.2+0.2+0.3+0.3=1$

## ECS 315: In-Class Exercise \# 4

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: $\mathbf{3 0} / \underline{\mathbf{0 8} / 2018}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| Name |  |  |  |
| Prapun | 5 | 5 | $\mathbf{5}$ |
|  |  |  |  |
|  |  |  |  |

In each of the parts below, find $P(A), P(B)$, and $P(A \cap B)$.
(a) $P\left(A^{c}\right)=0.5, P\left(B^{c}\right)=0.7$, and $P(A \cup B)=0.6$.

$$
\begin{aligned}
& P(A)=1-P\left(A^{c}\right)=1-0.5=0.5 \\
& P(B)=1-P\left(B^{c}\right)=1-0.7=0.3
\end{aligned}
$$

$$
\operatorname{From}(5.16), P(A \cup B)=P(A)+P(B)-P(A \cap B)
$$

Therefore, $\quad P(A \cap B)=P(A)+P(B)-P(A \cup B)=0.5+0.3-0.6=0.2$

$$
P(A)=0.5, P(B)=0.3, \text { and } P(A \cap B)=0.2
$$

(b) $P\left(A^{c} \cap B^{c}\right)=0.1, P\left(A \cap B^{c}\right)=0.2$, and $P\left(A^{c} \cap B\right)=0.3$.


We know that $P(\Omega)=1 \quad: P(A)=P\left(A \cap B^{c}\right)+P(A \cap B)$
Here, we must have
$0.1+0.2+P(A \cap B)+0.3=1$.
Therefore, $P(A \cap B)=0.4$
$=0.2+0.4$
$=0.6$
$P(B)=P(A \cap B)+P\left(A^{C} \cap B\right)$
$=0.4+0.3$
$=0.7$

$$
P(A)=0.6, P(B)=0.7, \text { and } P(A \cap B)=0.4
$$

(c) $P(A \cup B)=0.43, P\left(A \cup B^{c}\right)=0.62, P\left(A^{c} \cup B\right)=0.97$.
$P\left(A^{c} \cap B^{c}\right)=1-P(A \cup B)=1-0.43=0.57$
$P\left(A^{c} \cap B\right)=1-P\left(A \cup B^{c}\right)=1-0.62=0.38$
$P\left(A \cap B^{C}\right)=1-P\left(A^{C} \cup B\right)=1-0.97=0.03$


## ECS 315: In-Class Exercise \# 4 Alternative Solution

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: $\mathbf{3 0} / \underline{\mathbf{0 8} / 2018}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| Name | ID |  |  |
| Prapun | $\mathbf{5}$ | $\mathbf{5}$ | $\mathbf{5}$ |
|  |  |  |  |
|  |  |  |  |

In each of the parts below, find $P(A), P(B)$, and $P(A \cap B)$.
(a) $P\left(A^{c}\right)=0.5, P\left(B^{c}\right)=0.7$, and $P(A \cup B)=0.6$.

(b) $P\left(A^{c} \cap B^{c}\right)=0.1, P\left(A \cap B^{c}\right)=0.2$, and $P\left(A^{c} \cap B\right)=0.3$.


$$
\begin{aligned}
& P_{1}=0.1 \\
& p_{2}=0.2 \\
& p_{4}=0.3 \\
& p_{1}+p_{2}+p_{3}+p_{4}=1 \Rightarrow p_{3}=1-0.1-0.2-0.3=0.4
\end{aligned}
$$

$$
\begin{array}{cc}
p_{2}+p_{3}, & p_{3}+p_{4} \\
P(A)= \\
\hline 0.6, P(B)=\underline{0.7}, \text { and } P(A \cap B)=\underline{0.4}
\end{array}
$$

(c) $P(A \cup B)=0.43, P\left(A \cup B^{c}\right)=0.62, P\left(A^{c} \cup B\right)=0.97$.


$$
\left.\begin{array}{l}
p_{2}+p_{3}+p_{4}=0.43 \\
p_{1}+p_{2}+p_{3}=0.62 \\
p_{1}+p_{3}+p_{4}=0.97 \\
p_{1}+p_{2}+p_{3}+p_{4}=1
\end{array}\right\} \begin{aligned}
& p_{1}=0.57 \\
& p_{2}=0.03 \\
& p_{3}=0.02 \\
& p_{4}=0.38
\end{aligned}
$$

$$
P(A)=\begin{gathered}
P_{2}+P_{3}, P_{3}+P_{4} \\
0.05,
\end{gathered}, P(B)=\begin{aligned}
& P_{3} \\
& 0.40
\end{aligned}, \text { and } P(A \cap B)=\underline{0.02} .
$$

## ECS 315: In-Class Exercise \# 5

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: $\mathbf{3 0} / \underline{\mathbf{0}} / 2018$ |  |  |  |
| :--- | :--- | :--- | :--- |
| Name | ID |  |  |
| Prapun | $\mathbf{5}$ | $\mathbf{5}$ | $\mathbf{5}$ |
|  |  |  |  |
|  |  |  |  |

1. Calculate the following quantities:
a. $3!=3 \times 2 \times 1=6$
b. $\binom{6}{3}=\frac{6!}{3!3!}=\frac{6 \times 5 \times 4}{3 \times 2 \times 1}=20$
c. $(6)_{3}=6 \times 4 \times 5=120$
d. $\binom{6}{1,2,3}=\frac{6!}{1!2!3!}=\frac{3 \times 5 \times 4}{2}=60$
2. Suppose we sample 4 objects from a collection of 6 distinct objects.

Calculate the number of different possibilities when
a. the sampling is ordered and performed with replacement

$$
6 \times 6 \times 6 \times 6=6^{4}=1,296
$$

b. the sampling is ordered and performed without replacement

$$
6 \times 5 \times 4 \times 3=360
$$

c. the sampling is unordered and performed without replacement

$$
\binom{6}{4}=\frac{6!}{4!2!}=\frac{6 \times 5}{2}=15
$$

3. Calculate the number of different results when we permute
a. ABC

$$
3!=3 \times 2 \times 1=6
$$

b. AABBCC

$$
\frac{6!}{2!2!2!}=\frac{6 \times 5 \times 4 \times 3}{2 \times 2}=90
$$

Don't forget to simplify your answers.

## ECS 315: In-Class Exercise \# 6

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

1) Consider a random experiment whose sample space is $\{a, b, c, d\}$
with outcome probabilities $0.2,0.2,0.3$, and 0.3 , respectively.
Let $A=\{a, b, c\}, B=\{c, d\}$, and $C=\{a, c\}$.
Find the following probabilities. $A \cap B=\{c\}$

$$
A^{c} \cap B=B \backslash A=\{d\}
$$

$$
\begin{array}{|l|l|}
\hline P(A \mid B)=\frac{P(A \cap B)}{P(B)}=\frac{0.3}{0.6}=\frac{1}{2}=0.9 \\
P(B)=P(\{c, d\})=P(\{c\})+P(\{d\})=0.3+0.3=0.6
\end{array} \quad \begin{aligned}
& P\left(A^{c} \mid B\right)=\frac{P\left(A^{c} \cap B\right)=P(\{d\})}{P(B)}=\frac{0.3}{0.6}=\frac{1}{2}=0.5 \\
& \begin{array}{rl}
P\left(B^{c}\right)=1-P(B)=1-0.6=0.4 \\
P\left(A \mid B^{c}\right)=\frac{P\left(A \cap B^{c}\right)}{P\left(B^{c}\right)}=\frac{P(\{0, b\})}{0.4}=\frac{0.4}{0.4}=1 & P(A \cap B \mid C)=\frac{P(A \cap B \cap C)}{P(C)}=\frac{P(\{c\})}{0.2+0.3}=\frac{0.3}{0.5}=\frac{3}{5} \\
A \cap B^{c}=A \mid B=\{a, b\}
\end{array} \\
& =0.6
\end{aligned}
$$

2) Consider the following sequences of 1 s and 0 s which summarize the data obtained from 16 testees in a disease testing experiment.


$$
|\Omega|=16
$$

The results in the $i$-th column are for the $i$-th testee. The $D$ row indicates whether each of the testees actually has the disease under investigation. The TP row indicates whether each of the testees is tested positive for the disease.
Numbers " 1 " and " 0 " correspond to "True" and "False", respectively.
Suppose we randomly pick a testee from this pool of 16 persons. Let $D$ be the event that this selected person actually has the disease. Let $T_{P}$ be the event that this selected person is tested positive for the disease.
Find the following probabilities. No explanation is needed here.
$\left.\begin{array}{|l|l|l|}\hline & \text { Among the 16 testees, } \\ 9 \text { have the disease. }\end{array} \quad P(D)=\frac{9}{16} \quad P\left(T_{P}\right)=\frac{7}{16} \quad \begin{array}{l}\text { Among the } 16 \text { testees, } \\ 7 \text { test positive. }\end{array}\right]$

## ECS 315: In-Class Exercise \#

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

| Date: $1 \underline{1} / \underline{\mathbf{0}} / 2018$ |  |  |  |
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| Name | ID |  |  |
| Prapun | 5 | 5 | 5 |
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|  |  |  |  |

$$
P(H I V)=1 / 10=0.1
$$

Suppose that for the Land of $\mathrm{Oz}, 1$ in 10 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive ( + ) or negative (-) response. Suppose the test gives the correct answer $90 \%$ of the time. (The test is $90 \%$ accurate.)
(a) What is $\mathrm{P}(-\mid \mathrm{HIV})$, the conditional probability that a person tests negative given that the person does have the H/V virus?

The test gives incorrect result.

$$
P(-\mid H I V)=1-P(+\mid H I V)=1-0.9=0.1
$$

(b) Find the probability that a randomly chosen person tests positive.

$$
\begin{aligned}
P(+) & =P(+\mid H I V) P(H I V)+P\left(+\mid H I V^{c}\right) P\left(H I V^{c}\right) \\
& =0.9 \times 0.1+\quad 0.1 \times(1-0.1) \\
& =0.09 \times 2=0.18
\end{aligned}
$$

(c) Find the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive.

$$
P(H I V \mid+)=\frac{P(+\mid H I V) P(H I V)}{P(+)}=\frac{0.9 \times 0.1}{0.18}=\frac{1}{2}=50 \%
$$

ECS 315: In-Class Exercise \# 8

Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Date: 18/09 / 2018
Name

Prapun
3. Do not panic.
(1) Consider events A, B, C, D defined on a sample space $\Omega$. Suppose

$$
\begin{gathered}
P(B)=1 / 3, P(D)=1 / 4 \\
P(A \mid B)=1 / 5, P\left(A \mid B^{c}\right)=3 / 5, P(A \mid D)=1
\end{gathered}
$$

(a) Find $P(A \cap B)$.

$$
P(A \cap B)=P(A \mid B) P(B)=\frac{1}{5} \times \frac{1}{3}=\frac{1}{15}
$$

(b) Use the total probability theorem to find $\mathrm{P}(A)$.

$$
P(A)=P(A \mid B) P(B)+P\left(A \mid B^{C}\right) \overbrace{P\left(B^{C}\right)}^{1-P(B)=1-\frac{1}{3}=\frac{2}{3}} \times \frac{1}{3}+\frac{3}{5} \times \frac{2}{3}=\frac{1}{15}+\frac{6}{15}=\frac{7}{15}
$$

(c) Find $P(B \mid A)$. Bayes' theorem (form $x 1$ )

$$
P(B \mid A)=\frac{P(A \mid B) P(B)}{P(A)}=\frac{\frac{1}{5} \times \frac{1}{3}}{\frac{7}{15}}=\frac{1}{7}
$$

(d) Find $P\left(A \mid D^{c}\right)$. Again, we apply the total probability theorem

Method 1:

$$
\begin{aligned}
P(A) & =P(A \mid D) P(D)+P\left(A \mid D^{c}\right) P\left(D^{c}\right) \\
\frac{7}{15} & =1 \times \frac{1}{4}+P\left(A \mid D^{c}\right)\left(1-\frac{1}{4}\right) \\
P\left(A \mid D^{c}\right) & =\left(\frac{7}{15}-\frac{1}{4}\right) \times \frac{4}{3}=\frac{13}{50} \times \frac{41}{3}=\frac{13}{45}
\end{aligned}
$$

Method 2:
(2) Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$. Find $P(A \cap B)$ to make A and B /independent.

$$
\begin{aligned}
& P\left(A \mid D^{c}\right)=\frac{P\left(A \cap D^{c}\right)}{P\left(D^{C}\right)}=\frac{13 / 60}{3 / 4}=\frac{13}{45} \\
& P\left(A \cap D^{c}\right)=P(A)-P(A \cap D)=\frac{7}{15}-\frac{1}{4}=\frac{13}{60}
\end{aligned}
$$

To make $A \Perp B$, we need $P(A \cap B)=P(A) P(B)$
Therefore, $P(A \cap B)=\frac{1}{2} \times \frac{1}{3}=\frac{1}{6}$.


$$
P(A \cap D)=P(D) P(A \mid D)=\frac{1}{4} \times 1=\frac{1}{4}
$$

(3) Suppose $P(C)=\frac{1}{3}$ and $P\left(D \cap C^{c}\right)=\frac{1}{6}$. Find $P(C \cap D)$ to make $C$ and $D$ independent.

To make $C \| D$, we need $P(C \cap D)=P(C) P(D)$. Let $P(C \cap D)=$ or.


Alternatively, Another equivalent property $x=\frac{1}{6}$
we need $P\left(D \cap C^{c}\right) \stackrel{\downarrow}{=} P(D) P\left(C^{c}\right)$

$$
\frac{1}{6}=P(D)\left(1-\frac{1}{3}\right) \Rightarrow P(D)=\frac{1}{6} \times \frac{3}{2}=\frac{1}{4} \Rightarrow P(C \cap D)=P(C) P(D)=\frac{1}{3} \times \frac{1}{4}=\frac{1}{12}
$$

## ECS 315: In-Class Exercise \# 9

## Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Do not panic.

1) [Digital Communication] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.4. Assume bit errors occur independently. Your answers for parts (a) and (b) should be of the form X.XXXX.
a) Suppose we input bit sequence " 10101010 " into this channel.
i) What is the probability that the output is " 1000001 "?

$$
(1-p) \times(1-p) \times p \times(1-p) \times p \times(1-p) \times(1-p)=(1-p)^{5} p^{2}=0.6^{5} \times 0.4^{2}=\frac{972}{78125} \approx 0.0124
$$

ii) What is the probability that exactly 4 bits are in error at the channel output?

$$
\binom{7}{4} p^{4}(1-p)^{3}=\frac{7 \times 6 \times 5}{3 \times 2 \times 1} 0.4^{4} 0.6^{3}=0.1935
$$

iii) What is the probability that there is at least one bit error at the channel output?

$$
1-(1-p)^{7}=1-0.6^{7} \approx 0.9720
$$

b) Suppose we keep inputting bits into this channel. What is the probability that the first bit error at the output occurs on the fourth bit?

$$
(1-p)^{3} p=0.6^{3} \times 0.4=0.0864
$$

c) (Optional) Suppose the input bits are generated by flipping a fair coin 7 times. Heads and tails are represented by 1 and 0 , respectively.

Let $A$ be the event that the output of the channel is " 1000001 ".
Let $B_{1}$ be the event that the input of the channel is "1100011". $\Rightarrow P\left(A \mid B_{1}\right)=p^{2}(1-p)^{5}$
Let $B_{2}$ be the event that the input of the channel is "1011101". $\Rightarrow P\left(A \mid B_{2}\right)=p^{3}(1-p)^{4}$
Compare $P\left(B_{1} \mid A\right)$ and $P\left(B_{2} \mid A\right)$. (Which one is larger? Explain.)

$$
\begin{aligned}
& P\left(B_{1} \mid A\right)= \frac{P\left(B_{1} \cap A\right)}{P(A)}=\frac{P\left(A \mid B_{1}\right) P\left(B_{1}\right)}{P(A)} \quad \text { Note } \\
& P\left(B_{2} \mid A\right)= \frac{P\left(B_{2} \cap A\right)}{P \mid A)}=\frac{P\left(A \mid B_{2}\right) P\left(B_{2}\right)}{P(A)} \\
&(* *) \\
& \quad \text { Hen } \\
& \begin{array}{l}
P\left(B_{1} \mid A\right) \downarrow P\left(A \mid B_{1}\right) \downarrow \\
P\left(B_{2} \mid A\right) \\
P\left(A \mid B_{2}\right) \\
P^{2}(1-P)^{5} \\
P^{3}(1-P)^{4}
\end{array} \frac{1-P}{P}=\frac{6}{4}>1 \\
& \quad \Rightarrow P\left(B_{1} \mid A\right)>P\left(B_{2} \mid A\right)
\end{aligned}
$$

